

# Complementarity as corollary

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Recently, Bohr's complementarity principle was assessed both theoretically and experimentally in setups involving delayed choices. These works argued in favor of a reformulation of the aforementioned principle so as to account for situations in which quantum systems would behave simultaneously as wave and particle. Here we defend a framework that, supported by well-known experimental results and consistent with the decoherence program, allows us to interpret complementarity in terms of correlations of the system with an *informer*. Our proposal offers an operational interpretation for the wavelike behavior both in terms of nonlocal resources and the couple work-information. Moreover, our results invite us to reconsider the current status of the complementarity principle.

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Bohr's complementarity [1] is widely accepted as a fundamental feature of quantum mechanics. As such, it is often taken as a separate postulate, despite the fact it lacks a consensual formulation. While some works associate the notion of complementarity with noncommuting observables [2–4] others link it with mutually exclusive experimental setups. Within the latter context, complementarity was recently revisited, receiving both theoretical [5, 6] and experimental [7–9] assessments. Defining the notions of wave and particle in terms of the statistics of clicks in the detectors and using quantum beam splitters, these works investigated a quantum version of Wheeler's delayed-choice experiment (DCE) [10, 11] and arrived at the conclusion that the complementarity principle has to be updated in order to account for a *morphing* behavior.

The argument put forward in Refs. [5–12] is constructed as follows. A generic quantum system, hereafter called *quanton*, impinges on a Mach-Zehnder interferometer (MZI) [Fig. 1(a)] through a path  $|0\rangle$ . After being split by  $BS_1$  in a superposition of distinguishable paths and receiving a relative phase, the quanton ends up in  $|p\rangle = (|0\rangle + ie^{i\varphi}|1\rangle)/\sqrt{2}$ . If  $BS_2$  is absent, the detectors randomly click with probability  $\frac{1}{2}$ . Since the arm traveled is assumed to be revealed upon a click,  $|p\rangle$  is viewed as a *particlelike* state, what justifies the label “*p*”. On the other hand, being present,  $BS_2$  recombines the amplitudes and yields  $|w\rangle = \cos(\frac{\varphi}{2})|1\rangle - \sin(\frac{\varphi}{2})|0\rangle$ , up to a global phase. Given that the statistics is now sensitive to the phase  $\varphi$ , the quanton is meant to travel both arms simultaneously, just like a *wave*.

By delaying the choice of inserting  $BS_2$ , Wheeler's proposal aims to defy the assumption according to which some hidden variable would let the quanton know about the state of  $BS_2$ . Having been “informed” about the absence of  $BS_2$ , the quanton would choose one route, so that it could no longer produce interference even if  $BS_2$  were suddenly inserted. Recently, however, such a DCE

was realized and interference was observed [12], in accordance with the statistics associated with  $|w\rangle$ . Wheeler would have interpreted this result as follows [10]: “*Does this result mean that present choice influences past dynamics, in contravention of every formulation of causality? Or does it mean, calculate pedantically and don't ask questions? Neither; the lesson presents itself rather like this, that the past has no existence except as it is recorded in the present.*”

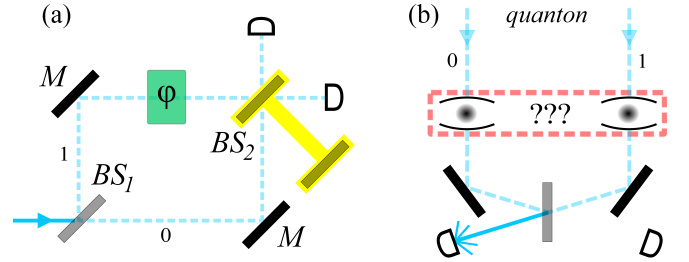


FIG. 1. (Color online) (a) Mach-Zehnder interferometer composed of two mirrors ( $M$ ), two beam splitters ( $BS_{1,2}$ ), a tunable phase shifter, and two detectors. The quantum setup is implemented by letting  $BS_2$  be in superposition. (b) “Wave detector.” A quanton is sent towards two possible paths 0 and 1, nondestructively interacts with two trapped qubits, is redirected by two mirrors towards a beam splitter, and finally clicks a detector. When the quanton state is not *strictly particlelike*, the two qubits get entangled after the click.

In the quantum version of the DCE [5–9],  $BS_2$  is prepared in the state  $\cos \alpha |\text{out}\rangle + \sin \alpha |\text{in}\rangle$ , a superposition of being *in* and *out* the interferometer [Fig. 1(a)]. The global state right before the clicks is then given by

$$|\psi\rangle = \cos \alpha |p\rangle |\text{out}\rangle + \sin \alpha |w\rangle |\text{in}\rangle, \quad (1)$$

from which it is concluded that the complementarity principle must be redefined, since a single setup has been exhibited in which both behaviors can coexist. The interpretation underlying this result differs from Wheeler's in essence: “*Behavior is in the eye of the observer*”, for

“particle and wave are not realistic properties but merely reflect how we look at the photon” [5].

By basing the diagnostic “wave or particle” on the resulting statistics, the above conceptual framework matches, by construction, Bohr’s conception of a “whole unit”, but disconnects the notions of wave and particle from the vector state and its time evolution. The point can be appreciated as follows. First, when we fix  $\varphi$ , it is not possible to infer the quanton behavior solely from  $|p\rangle$  and  $|w\rangle$ , while the query about the path the quanton takes keeps legitimate. In particular, if  $\varphi = \frac{\pi}{2}$ , then  $|p\rangle = |w\rangle$ . Second, and more important, in the region between the beam splitters, Schrödinger’s equation predicts that the state of the quanton is always  $|p\rangle$ . If the quanton behavior were to be inferred from  $|p\rangle$  one should arrive at the conflicting conclusion that the quanton travels like a particle even when  $BS_2$  is present. Third, the statistics does not distinguish between a superposition  $\sqrt{(1-x)}|0\rangle + e^{i\varphi}\sqrt{x}|1\rangle$ , which can encode a phase, from a *decohered* state  $(1-x)|0\rangle\langle 0| + x|1\rangle\langle 1|$ , which cannot. In addition, as remarked in Ref. [5], the approach is not able to shed light on a striking facet of duality which manifests in the “*tension between the observed interference and the detection of individual photons, one by one, by clicks in the detectors.*” In fact, since the detector is part of the “whole unit” that determines the behavior of the quanton, one would need another setup to qualify the behavior of the detector.

In this paper we advance a framework that overcomes the above difficulties and provides new insights on physical mechanisms underlying duality. We start by looking for an operational meaning for the concepts of wave and particle. Let us consider the “wave detector” depicted in Fig. 1(b). A quanton traveling the generic paths  $|0\rangle$  and  $|1\rangle$  interacts with two trapped qbits initially prepared in the ground state  $|0\rangle_{0,1}$ . The interaction is non-destructive, meaning that  $|0\rangle|0\rangle_0|0\rangle_1 \rightarrow |0\rangle|1\rangle_0|0\rangle_1$  and  $|1\rangle|0\rangle_0|0\rangle_1 \rightarrow |1\rangle|0\rangle_0|1\rangle_1$ . After being redirected by two mirrors towards a beam splitter, the quanton causes a click in one of the detectors. We call *strictly wavelike* those quanton states that, upon clicking a detector, leave the two qbits *maximally entangled*. Certainly, no entanglement can be produced if the quanton travels only one of the paths.

Suppose that a quanton (now a qbit) enters the machine of Fig. 1(b) in a Werner state,  $\rho_Q^W = (1-x)\frac{1}{2} + x|\psi\rangle\langle\psi|$ , where  $x \in [0, 1]$  and  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ , with  $|\alpha|^2 + |\beta|^2 = 1$ . Assume the initial state of the system,  $\rho_Q^W \otimes |0\rangle\langle 0|_0 \otimes |0\rangle\langle 0|_1$ , evolves to  $\tilde{\rho}$  when the quanton leaves the beam splitter. After a click in the detector  $k$  ( $k = 0, 1$ ), the conditional state of the qbits,  $\rho_{\text{qbits}|k} = \text{Tr}_Q(\Pi_k \tilde{\rho}) / \text{Tr}(\Pi_k \tilde{\rho})$ , is written as

$$\rho_{\text{qbits}|k} = \left(\frac{1-x}{2} + x|\alpha|^2\right)|10\rangle\langle 10| + \left(\frac{1-x}{2} + x|\beta|^2\right)|01\rangle\langle 01| \\ + (-1)^k (i x \alpha^* \beta |01\rangle\langle 10| - i x \alpha \beta^* |10\rangle\langle 01|).$$

Let  $\mathcal{N}_l := \max\{0, \frac{1}{4}B_{\max}^2 - 1\}$  be the maximum degree of

violation of a CHSH Bell-inequality achieved via an optimal set of measurements ( $B_{\max}$  defined in Refs. [13, 14]). For the above state, one shows that  $\mathcal{N}_l = 4x^2|\alpha\beta|^2$ , which is a measure of the nonlocality activated in the two-qbit system by the quanton. The entanglement produced in the machine,  $E = 2x|\alpha\beta|$ , can be computed via concurrence [15]. Thus, nonlocality and entanglement will be maximum only if  $x = 1$  and  $|\alpha| = |\beta| = 1/\sqrt{2}$ .

We see that input states like  $a|0\rangle\langle 0| + (1-a)|1\rangle\langle 1|$  ( $a \in [0, 1]$ ),  $|0\rangle$ , and  $|1\rangle$ , which do not have *coherences* in the pertinent basis, cannot activate nonlocality and, therefore, according to our criterion, are to be regarded as *strictly particlelike*. This point can be further elaborated as follows. Let  $\{|k\rangle\}$  be the eigenbasis of an observable  $\mathcal{K}$  spanning a Hilbert space  $\mathcal{H}_Q$  with dimension  $d_Q$ . Then, there holds that  $\Pi_k \Pi_{k'} = \Pi_k \delta_{k,k'}$  and  $\sum_k \Pi_k = 1$ , for projectors  $\Pi_k = |k\rangle\langle k|$ .  $\mathcal{K}$  is assumed to have a well defined classical interpretation, such as energy, position, or momentum. When a quanton is in an eigenstate  $|k\rangle$ , we know with certainty the outcome even before a measurement of  $\mathcal{K}$ , so that the quanton is in a *definite path*. The claim of definiteness for a mixed state can be understood as follows. Consider a situation in which Alice measures the observable  $\mathcal{K}$  and next deliveries the quanton to Bob, without telling him the outcome. Since Bob knows which observable was measured, he knows for sure that in each run of the task he receives a projected state  $\rho_k = \Pi_k \rho_Q \Pi_k / p_k$  with probability  $p_k = \text{Tr}(\Pi_k \rho_Q \Pi_k)$ . Clearly, Bob always receives a quanton in a definite path,  $\rho_k = \Pi_k$ . However, without accessing the outcomes, Bob’s prediction for the whole ensemble is a statistical mixture of definite paths,  $\sum_k p_k \rho_k$ . By use of the joint entropy theorem [16],  $S(\sum_k p_k \rho_k) = H(p_k) + \sum_k p_k S(\rho_k)$ , where  $S$  and  $H$  stand for the von Neumann and Shannon entropies, respectively, we see that  $S(\sum_k p_k \Pi_k) = H(p_k) = -\sum_k p_k \ln p_k$ . That is, Bob’s lack of knowledge is nothing but subjective ignorance associated with the classical probability distribution  $p_k$  secretly prepared by Alice.

These aspects naturally fit our classical intuition about the particlelike behavior, according to which a particle always is well localized, even when its path is subjectively ignored. Now, if a quanton is in a definite path, then its behavior cannot be disturbed by a projective measurement on that path. This motivates us to link the notion of *particle* with states that satisfy

$$\Pi[\rho_Q] = \rho_Q, \quad (2)$$

where  $\Pi[\rho_Q] := \sum_k \Pi_k \rho_Q \Pi_k$  describes the process of unread measurements discussed above. The scenario is clearly different for a “superposition of paths”, e.g.  $(|0\rangle + e^{i\varphi}|1\rangle)/\sqrt{2}$ , whose coherence is destroyed upon measurements of  $\mathcal{K}$ .

If a state  $\rho_Q$  is particlelike with regards to an observable  $\mathcal{K}$ , then it follows from (2) that  $f(\rho_Q) = f(\Pi[\rho_Q])$ , for any generic (to-be-specified)  $f$ . We then quantify the

extent to which a state  $\rho_Q$  is *wavelike* upon measurements of  $\mathcal{K}$  by

$$I_w(\rho_Q) := \text{Tr} \left[ f(\rho_Q) - f(\Pi[\rho_Q]) \right]. \quad (3)$$

This gives a generic measure between the state under analysis and a copy subjected to measurements of  $\mathcal{K}$ . For any *differentiable convex*  $f$  we can show, by use of the generalized Klein's inequality [16] and some results proved in Ref. [17], that  $0 \leq I_w(\rho_Q) \leq I_w^{ub}(\rho_Q)$ , where the upper bound is given by  $I_w^{ub}(\rho_Q) = \text{Tr}[(\rho_Q - \Pi[\rho_Q])f'(\rho_Q)]$ . If we assume, in addition, that  $f$  is *strictly convex*, then we get that  $I_w(\rho_Q) = 0$  iff  $\Pi[\rho_Q] = \rho_Q$ , i.e., if and only if the state is particlelike. Then, we associate  $f$  with measures of information ( $I$ ), generally given by convex functions. We take

$$\text{Tr} f(\rho_Q) = S_{\max} - S(\rho_Q) =: I(\rho_Q), \quad (4)$$

where  $S_{\max}$  is the maximum entropy in a given Hilbert space. A possible specification for the entropic principle in Eq. (4) is the Tsallis entropy [18],  $S_q(\rho) = \frac{1 - \text{Tr} \rho^q}{q-1}$  ( $q > 0 \in \mathbb{R}$ ), which is non-negative and strictly concave. It reduces to the von Neumann entropy as  $q \rightarrow 1$  and recovers the linear entropy  $S_2 = 1 - \text{Tr} \rho^2$  as  $q = 2$ . Then, Eq. (3) becomes  $I_w^{(q)}(\rho_Q) = S_q(\Pi[\rho_Q]) - S_q(\rho_Q)$ . For  $q = 2$ , a geometric measure derives which is rather convenient for computations [17]:

$$I_w^{(2)}(\rho_Q) = \|\rho_Q - \Pi[\rho_Q]\|^2, \quad (5)$$

where  $\|\rho\|^2 := \text{Tr}(\rho^\dagger \rho)$  is the square norm in the Hilbert-Schmidt space. By Eq. (4), the respective measure of information reads  $I^{(2)}(\rho_Q) = S_{2,\max} - S_2(\rho_Q)$ , with  $S_{\max} = 1 - d_Q^{-1}$ . On the other hand, the measure associated with  $q = 1$  offers conceptual advantage, as shown next. Adopting the von Neumann entropy makes the *wavelike information*,  $I_w^{(q)}$ , reduce to

$$I_w(\rho_Q) = S(\Pi[\rho_Q]) - S(\rho_Q). \quad (6)$$

The superscript “(1)” will be omitted when  $q = 1$ . Remarkably,  $I_w$  possesses a direct thermodynamic interpretation. The key point behind this idea is the link work-information,  $W(\rho_Q) = k_B T I(\rho_Q)$  [19, 20], where  $k_B$  is the Boltzmann constant and  $I$  is defined in terms of the von Neumann entropy. This relation gives the amount of work  $W$  one can draw from a heat bath of temperature  $T$  by use of the state  $\rho_Q$ . Along with Eq. (4), it allows us to express the wavelike information as

$$k_B T I_w(\rho_Q) = W(\rho_Q) - W(\Pi[\rho_Q]), \quad (7)$$

whose interpretation is as follows. Suppose that Alice prepares a state  $\rho_Q$  and delivers the quanton to Bob, who can extract an amount  $W(\rho_Q)$  of work from the heat bath. In a second scenario, the delivered quanton is intercepted by a *classical demon* [19], who measures  $\mathcal{K}$  and

forwards the quanton to Bob. In this case, Bob can extract only an amount  $W(\Pi[\rho_Q])$  of work. The wavelike information  $I_w$  turns out to be directly related to the difference of work that Bob can extract by using the inviolate state  $\rho_Q$  sent by Alice and a counterpart secretly accessed by a local Maxwell's demon. Being particlelike  $\rho_Q$  offers no advantage in relation to its measured counterpart.

It is instructive to apply our measure (6) to some noticeable states. For a pure state  $|\psi\rangle$ ,  $I_w = H(p_k)$ , where  $p_k = |\langle k|\psi\rangle|^2$ . In particular, for  $|\psi\rangle = \frac{1}{\sqrt{n}} \sum_k |k\rangle$ , with  $n \leq d_Q$ , one has that  $I_w = \ln n$ . Clearly, the wavelike information increases with the number  $n$  of branches in the superposition and vanishes for a definite path. For the states  $|p\rangle$  and  $|w\rangle$  one shows that  $I_w(|p\rangle) = \ln 2$  and  $I_w(|w\rangle) = -x \ln x - y \ln y$ , with  $x = 1 - y = \cos^2(\varphi/2)$ . Since  $I_w(|p\rangle) \geq I_w(|w\rangle)$ , we may say, in dissonance with Refs. [5–9], that  $|p\rangle$  is more wavelike than  $|w\rangle$ . Finally, it is interesting to check whether our measure is consistent with the diagnostic provided by our wave detector [Fig. 1(b)]. The wavelike information for the Werner state gives  $I_w^{(2)}(\rho_Q^W) = 2x^2|\alpha\beta|^2 = 2\mathcal{N}$ , which formally shows that nonlocality can be activated in the machine only if the input state has some wavelike character.

We now discuss a second defining feature of our approach, whereby we aim to ponderate on how waves and particles, as defined in the present context, are produced in nature. To this end, we retrieve an idea according to which particlelike behavior appears in the presence of some extra degree of freedom which, being correlated with the quanton, can furnish which-path information. For this reason, we call such a degree of freedom an *informer*. This viewpoint was theoretically identified [21–23] and experimentally demonstrated [24–26] sometime ago. Here, however, we aim to advance the more radical position that takes that mechanism as *the* physical principle underlying duality.

To elaborate on this view we should first note that whatever the quanton state, we can always conceive a purification  $|\Psi\rangle \in \mathcal{H}_Q \otimes \mathcal{H}_I$ , where  $\mathcal{H}_I$  stands for the Hilbert space of the informer and  $\rho_Q = \text{Tr}_I |\Psi\rangle\langle\Psi|$ . Note that, in general,  $\mathcal{H}_I$  may be itself a multipartite Hilbert space. It immediately follows that  $S(\rho_Q) = E(|\Psi\rangle)$ , the amount of entanglement in  $|\Psi\rangle$ . In search of a *complementary* formulation, we rewrite Eq. (6) as

$$I_w(\rho_Q) + I_p(\rho_Q) = I_{\mathcal{H}_Q}, \quad (8)$$

where  $I_{\mathcal{H}_Q} = \ln d_Q$  gives the maximum information available in the Hilbert space of the quanton, and

$$I_p(\rho_Q) := I(\Pi[\rho_Q]) + E(|\Psi\rangle). \quad (9)$$

This expression defines the *particlelike information*, whereas  $I(\Pi[\rho_Q]) = \ln d_Q - S(\Pi[\rho_Q])$  can be interpreted as the amount of information accessible via unread measurements of  $\mathcal{K}$ . Equations (6)–(9) give an insightful pic-

ture of complementarity. First, we see that the particlelike behavior is determined by both the measurement-accessible information and the entanglement with the informer. Second,  $I_p$  can saturate due to entanglement, a measurement-independent correlation. When the entanglement with the informer is maximum ( $E = \ln d_Q$ ), then  $I_w = 0$  and  $I_p = \ln d_Q$ , meaning that the quanton has a strictly particlelike behavior regardless the observable one chooses to make the diagnostic. Third, by focusing the analysis on the state of the quanton, we are able to describe the quanton behavior at any instant of time, automatically obeying the causal evolution prescribed by Schrödinger's equation. Any change in the quanton behavior is, therefore, constrained to physical interactions modeled in the Hamiltonian of the system and any mysterious role of the “whole unit” can be physically explained. In particular, we conclude that in the MZI the quanton always travels the arms in a strictly wavelike manner, as  $I_w(|p\rangle) = \ln 2$  regardless the presence of  $BS_2$ . Fourth, our approach can shed some light on the problem of individual clicks in the detectors. To better illustrate the point, let us consider a minimum model of measurement.

Suppose a quanton is prepared in a wavelike state,  $|\mathcal{Q}\rangle = \sum_k c_k |k\rangle$ , for which  $I_w(|\mathcal{Q}\rangle) = H(|c_k|^2)$ . Consider an apparatus (informer) prepared in  $|\uparrow\rangle$ . After its interaction with the quanton, say via the mapping  $|k\rangle|\uparrow\rangle \mapsto |k\rangle|\uparrow_k\rangle$ , the system state becomes  $\sum_k c_k |k\rangle|\uparrow_k\rangle$ , where  $\{|\uparrow_k\rangle\}$  is an orthonormal basis in  $\mathcal{H}_I$ . In a measurement, we never directly look at the quanton. The macroscopic pointer has to be consulted (e.g., by means of scattered photons) and, at this instant, the collapse—a Bayesian updating of information—occurs. The reader of the pointer, say Alice, then obtains information about the quanton. Upon reading the outcome  $\uparrow_k$ , Alice concludes that both the pointer and the quanton are particlelike, as  $I_w(|k\rangle) = I_w(|\uparrow_k\rangle) = 0$ . A second observer, Bob, who is aware of the physical interaction between the quanton and the apparatus but does not read the pointer, cannot “collapse” the state. Still, by tracing out one of the partitions, Bob as well concludes that both the quanton and the pointer are particlelike:  $I_w(\rho_{Q,I}) = 0$ . This analysis illustrates how the wavelike information of the quanton changes in a measurement process while the pointer preserves its particlelike identity, thus explaining the individual clicks. Note that the inclusion of an environment would neither significantly contribute to this model nor change the conclusion.

Now we can conclude the analysis on the genesis of the dual behavior. Consider an entangled state  $\sum_k c_k |k\rangle|\mathcal{I}_k\rangle$ , where  $|\mathcal{I}_k\rangle \in \mathcal{H}_I$ . The reduced state reads  $\rho_Q = \sum_{kk'} c_k c_{k'}^* \langle \mathcal{I}_{k'} | \mathcal{I}_k \rangle |k\rangle \langle k'|$ . Since  $I_w(\rho_Q) = 0$  iff  $\rho_Q = \sum_k p_k \Pi_k$ , strictly particlelike behavior will occur for the state under inspection only if  $\langle \mathcal{I}_{k'} | \mathcal{I}_k \rangle = \delta_{k',k}$ . In this case, the information of every quanton state  $|k\rangle$  is encoded in a unique informer state  $|\mathcal{I}_k\rangle$ . From the viewpoint of the informer, therefore, the situation is such

that the quanton is always in a definite path. Nevertheless, there is no further informer witnessing the path of the informer itself, so that there is still a lack of information for the joint state. This explains how entanglement makes the quanton path definite while the joint state remains in a superposition of paths. On the other hand, given that  $\rho_Q = \sum_k p_k \Pi_k$  one can build a purification  $\sum_k c_k |k\rangle|\uparrow_k\rangle$ , with  $p_k = |c_k|^2$ , showing that particlelike behavior can always be thought of as emerging due to entanglement with an informer (*decoherence*). If entanglement is absent, genuine particlelike behavior appears only when the quanton is in an eigenstate of  $\mathcal{K}$ . However, as we have seen in our measurement model, as far as information is concerned a collapsed state  $|k\rangle|\uparrow_k\rangle$  from Alice's perspective is physically equivalent to a noncollapsed entangled state  $\sum_k c_k |k\rangle|\uparrow_k\rangle$  from Bob's perspective, provided  $|\uparrow\rangle$  is a sensorially accessible pointer. Once again, entanglement is found to be crucial. The wavelike behavior, by its turn, is favored in the following situation. Let  $\gamma_Q + \gamma_I = \Gamma$  be a generic conservation law. If the Hamiltonian obeys this law, then particlelike states such as  $|\gamma_Q\rangle|\gamma_I\rangle$  may evolve to  $\alpha|\gamma_Q\rangle|\gamma_I\rangle + \beta|\gamma_Q + \delta\gamma\rangle|\gamma_I - \delta\gamma\rangle$ , which yields particlelike reduced states. However, when  $\gamma_Q \ll \delta\gamma \ll \gamma_I$  the situation is such that the informer cannot be significantly disturbed upon interactions with the quanton. It follows that the final state can be approximated by  $(\alpha|\gamma_Q\rangle + \beta|\gamma_Q + \delta\gamma\rangle)|\gamma_I\rangle$ , in which case one has that  $I_w^{(2)}(\rho_Q) = 2|\alpha\beta|^2$ . Strictly wavelike behavior emerges for  $|\alpha| = |\beta| = 1/\sqrt{2}$ . This corresponds to the case of a quanton passing through a beam splitter or a double-slit system: A momentum-conserving interaction of a quanton with an undisturbable object makes the former behave like a wave. This scenario prompts us to propose the following primitive statement for duality: *Genuine particlelike behavior can emerge only when the quanton is entangled with an informer.* It is worth noting that likewise Englert's approach [22], ours admits both the fundamental role of the informer and the occurrence of a *morphing* behavior, here signaled when  $I_w > 0$  and  $I_p > 0$  simultaneously. As advantages, our formulation for duality relies on an equality [Eq. (8)] and directly applies to systems with arbitrary dimension.

As far as DCEs are concerned, within our framework the logic of “delaying a choice” makes no sense. Immediately after  $BS_1$ , the quanton can never behave as a particle, as there is no informer in the setup. The posterior introduction of  $BS_2$  cannot change the past. Upon interacting with  $BS_2$  the quanton behavior causally changes. By tracing  $BS_2$  out from state (1) we can directly compute  $I_p^{(2)}(\rho_Q) = \frac{1}{2}(1 - \cos^4 \alpha) \cos^2 \varphi$ , for the particlelike information, and  $E^{(2)} = \frac{1}{4} \sin^2(2\alpha) \cos^2 \varphi$ , for the entanglement as measured via linear entropy. If  $BS_2$  is in superposition ( $\alpha \neq 0, \frac{\pi}{2}$ ), then  $I_p^{(2)}(\rho_Q)$  cannot reach the maximum value  $\frac{1}{2}$  and hence the quanton can



never behave as a particle before reaching the detectors. Morphing behavior already appears in the classical MZI ( $\alpha = \frac{\pi}{2}$ ), where  $I_w^{(2)}$  and  $I_p^{(2)}$  can be easily adjusted by  $\varphi$ .

Finally, one should stress that information, as measured by  $I_{w,p}$ , is *relational*, i.e., it depends on the *observable* one uses to probe (*relate to*) it. For instance, the eigenstate  $|0\rangle$  of  $\sigma_z$  will be diagnosed as particlelike upon measurements of  $\sigma_z$  and as wavelike upon measurements of  $\sigma_x$ . This has to be so, as the capability of the state  $|0\rangle$  to activate nonlocality (in our wave detector) crucially depends on how we orientate the Stern-Gerlach in order to previously separate the paths. The “relationality of information” clearly derives from the noncommutability of the observables involved. This can be formally expressed by  $\mathcal{J} - \Pi[\mathcal{J}] = \sum_k [\mathcal{J}, \Pi_k] \Pi_k$ , which shows that an arbitrary operator  $\mathcal{J}$  will not be disturbed upon measurements of  $\mathcal{K}$  if and only if it commutes with the latter. This remark illustrates how our approach links to the issue of the complementarity of noncommuting observables. In another vein, one may wonder whether information is also *relative*, i.e., if for a given reference observable, distinct *observers* would access different amounts of  $I_{w,p}$ , while predicting the same physical fact (interference). This problem was recently investigated in the context of quantum references frames [27]. By considering a double-slit experiment inside a very light lab whose movement would constitute an informer for an external observer, and adopting a relative-state formulation, the authors have discussed how the same physical fact manifests in the viewpoint of each observer. Their approach and conclusions are consistent with the primitive elements adopted here for duality. Nevertheless, to the best of our knowledge, a relative-information approach to this problem still waits to be accomplished.

In summary, we have proposed an interpretation for the wave-particle duality that explains all experimental results and avoids conceptual puzzles. Our interpretation abdicates Bohr’s notion of a “whole unit” in favor of an informational substance for the vector state. This choice reveals to be rather fruitful: Besides offering a clear connection with thermodynamic work and nonlocality, it allows for consistently modeling the problem of individual clicks in the detectors. Also, our approach states duality in terms of primitive elements of the quantum theory, namely, deterministic evolution (Schrödinger’s equation), physical causation (interactions), correlations (the role of the informer), and the partial trace (which enables the diagnostic of subsystems). Our proposal, therefore, gives a conceptual framework that opens the possibility for one to regard Bohr’s complementarity no longer as a *principle*, but as a *corollary*.

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